Portfolios in a Regime Shifting Non-Normal World:

Are Alternative Assets Beneficial?

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ABSTRACT

Adding five alternative assets to equity and bond portfolios is harmful for US investors. We use nineteen portfolio models in conjunction with dummy variable regression, and measure out-of-sample performance by both certainly equivalent ratios and Sharpe ratios. The presence of harmful diversification is robust to different estimation periods and levels of risk aversion, and to the use of two regimes. Harmful diversification is not primarily due to transactions costs or non-normal returns, but to estimation risk. Large estimation errors during the credit crisis (2007-09) account for the harmful diversification of three of the five alternative assets over the 1997-2015 period. (100 words)

Key words: alternative assets, diversification, estimation errors, transactions costs, non-normality, regimes, credit crisis

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1. Introduction

In recent decades there has been a considerable increase in investment in alternative assets, which is forecast to continue. As the growth rate in the value of investment in alternative assets has been much higher than the growth rate in the value of institutional portfolios, the relative importance of alternative assets in institutional portfolios has also increased. Pension plans are the largest group of institutional investors, and in 1995 alternative assets accounted for 5% of the portfolios of pension plans in Australia, UK, US, Canada, Japan, Netherlands and Switzerland (Towers Watson, 2015). By 2014 this percentage had grown to 25%, and for the US it had risen to 29%, making investment in alternative assets by US pension plans larger than in fixed income. In 2014 the 64 largest pension schemes and 20 largest sovereign wealth and public pension reserve funds in 36 OECD countries allocated 30.1% of their assets of $10.3 trillion to alternative and other assets, OECD (2016). In December 2015 Willis Towers Watson (2016) surveyed the 602 largest alternative asset managers globally and found they have $6.2 trillion invested in alternative assets (hedge funds, private equity, real estate, commodities, infrastructure, illiquid credit and real assets). These figures show that alternative assets have become as important as equities and fixed income in the portfolios of major institutional investors, and are seriously considered when making asset allocation decisions.

The main question we address is whether investing in alternative assets is beneficial for US investors. To this end we analyse the effects of adding five alternative assets to US equity and bond portfolios. Definitions of alternative assets differ, and we concentrate on five important types of alternative asset for which a time series of monthly data since 1994 is available - hedge funds, real estate, private equity commodities and emerging equity markets. While investment in alternative assets provides greater diversification, these assets may be subject to greater estimation errors, higher transactions costs and non-normal returns, leading to inferior out-of-sample portfolio performance. In which case diversification into alternative assets is harmful, rather than beneficial.

Diversification is a fundamental principle of finance and is generally accepted as offering a 'free lunch' of higher returns for a given level of risk, or lower risk for a given level of returns. As significant proportions of institutional portfolios are invested in alternative assets, the performance of portfolios diversified in this way is important. In the first study of the effects of adding two or more alternative assets to portfolios which uses out-of-sample tests and incorporates transactions costs; we show that diversification is harmful for the important real world case of alternative assets and US investors. We also investigate three possible explanations for this result - estimation errors, transactions costs and non-normality on our results.

We compare the out-of-sample performance of seven portfolios where zero, one and all five of the alternative assets are added to an equity and bond portfolio. These portfolios are formed using 19 portfolio models - eight types of mean-variance analysis, eight types of *ad hoc* model and three models which maximize a utility function, in conjunction with three levels of risk aversion. When estimating the portfolio input parameters we use both three year and four year rolling estimation periods, and also allow for regime switching in asset returns. We impose realistic bounds on the asset proportions, and incorporate transactions costs both when forming portfolios and when evaluating their performance. Out-of-sample performance is measured using certain equivalent returns (CERs), and Sharpe ratios. Because we have 19 portfolio techniques, in conjunction with seven combinations of assets, we are able to use dummy variable regression to estimate the differential portfolio performance due to adding alternative assets, while controlling for the influence of the 19 different portfolio models. We then investigate three possible explanations for this result - estimation errors, transactions costs and non-normality, and conclude that the major cause of harmful diversification is estimation errors. Allowance for the significant non-normality of alternative asset returns does not have a material effect on portfolio performance, and transactions costs are not an important cause of harmful diversification. We demonstrate that estimation errors in portfolio inputs are larger for most alternative assets than for equities and bonds, that these errors bias portfolio weights in the expected direction, and that the total size of these biases in asset weights increases as the number of assets in a portfolio is increased. Large estimation errors during the credit crisis are responsible for the harmful diversification of three of our five alternative assets - real estate, private equity and emerging markets; but not commodities and hedge funds.

Over twenty five studies have formed portfolios of equities, bonds and two or more of the five alternative assets we study; but only three of these studies investigate the out-of-sample benefits of including alternative assets when short sales are prohibited. Two of these studies find that diversification into alternative assets improves performance, but they have important differences from the current study. Neither paper incorporates transactions costs. In addition, Jackwerth and Slavutskaya (forthcoming) do not examine optimized portfolios, while the data period of Sa-Aadu et al (2010) excludes most of the credit crisis. As well as banning short sales and examining out-of-sample performance, the third study (Jacobs et al, 2014) allows for transactions costs. They find that diversification improves performance for three optimization techniques, but for another three optimization techniques diversification is harmful.

This is the first study to find clear empirical evidence of harmful diversification effects for alternative assets. We do this with no short sales and transactions costs are included both when forming and assessing portfolios. We ensure the robustness of our conclusions by using two different performance measures of out-of-sample returns (CERs and Sharpe ratios), a large number of portfolio models, three different levels of risk aversion and two lengths of the estimation window; and by investigating both one and two asset return regimes. This research is also the first to conduct a detailed empirical analysis of the causes of harmful diversification, and the first to engage in a detailed empirical analysis of the estimation errors associated with alternative assets.

Our data is described in Section 2, including an explanation of how the hedge fund returns were de-smoothed. This section also reports the correlations between our seven assets, and tests our assets returns for normality. In Section 3 we present the methodology we use when estimating the out-of-sample inputs to the portfolio models, followed by a summary of the 19 portfolio models and the constant relative risk aversion (CRRA) utility function we employ. Section 4 reports the transactions costs we use, and explains how these transactions costs are both included in the portfolio models and in the performance measures. An outline of the application of the Markov regime shifting model to our data, and the effect this has on the normality of returns appears in Section 5. Section 6 reveals how we measure performance using CERs and Sharpe ratios, and then has our results. Section 7 investigates the extent to which estimation errors, transactions costs and non-normality account for the harmful effects of diversification into alternative assets. Finally, Section 8 contains our conclusions.

2. Data and Descriptive Statistics

***2a. Data**.* We analyse monthly returns from January 1994 to December 2015 for five alternative assets - commodities (S&P GSCI total return index), real estate (FTSE EPRA/NAREIT global real estate total return index), private equity (LPX 50 listed private equity total return index), emerging markets (MSCI emerging markets total return index) and hedge funds (HFRI fund of funds composite total return index). The data for these five alternative assets are based on international portfolios of the underlying assets, and their returns are expressed in US dollars. We also analyse monthly returns for the same period on US equities (S&P 500 composite total return index) and US bonds (Barclays US aggregate total return index) in US dollars[1]. This bond index covers Treasury bonds, government-related securities, corporate bonds and securitizations. All the assets we study are exchange traded, except for hedge funds, which are not listed.

2b. De-smoothing Hedge Fund Returns. Hedge fund returns are based on non-market valuations of the underlying assets, which brings with it the problems of an illiquid market and managed returns, leading to smoothed and biased returns. The empirical evidence is that return smoothing creates positive serial correlation in returns, and reduces the variance and correlations with other assets (e.g. Agarawal et al, 2011; Cassar and Gerakos, 2011; Bollen and Pool, 2008, 2009). While many previous researchers have ignored this problem, some have addressed the smoothing of hedge index returns by transforming (de-smoothing) them to approximate the unsmoothed returns. Five methods for de-smoothing hedge fund returns have been used. The method of Geltner (1991, 1993) has been widely used for real estate returns, and applied to hedge funds by Bekkers et al (2009); Brooks and Kat (2002); and Hoevenaars et

[1] The private equity data came from Bloomberg, and the remaining data came from Datastream.

al (2008). Getmansky et al (2004) developed a method which they used to adjust hedge fund returns, and their method has also been used on hedge funds by Boigner, and Gadzinski (2015). Okunev and White (2004) adjusted hedge fund returns using their own method, which has also been used by Cavenaile et al (2011) and Bird et al (2013) for de-smoothing hedge find returns. Pedersen et al (2014) have used their own method for de-smoothing hedge fund returns, while the de-smoothing method of Bond et al (2007) has not been applied to hedge fund returns. In our analysis we use the Geltner (1991, 1993) method which is simple and widely used.

As well as return smoothing, the way hedge fund indices are constructed introduces various biases (e.g. survivorship bias, self-selection bias, and backfill bias) which may increase or decrease mean returns[2]. However Edelman et al (2013), Fung and Hsieh (2000, 2002, 2004), Ackermann et al (1999) and Pictet Alternative Advisors (2015) argue that these biases in hedge fund returns tend to be offsetting, and that the use of an index of funds of funds mitigates them even further. Because we are using a composite fund of funds index we do not adjust hedge fund returns for these biases.

2c. Normality Tests. Many previous studies have applied the Jarque-Bera normality test to asset returns and confirmed that alternative asset returns have non-normal distributions. Mean-variance portfolio analysis assumes either that returns are normally distributed, or that investors have a quadratic utility function. Since quadratic utility is subject to serious objections, the use of mean-variance analysis rests on the normality of asset returns. If this assumption is not met higher moments of the return distribution, such as skewness and kurtosis, may be important to investors. Table 1 shows the first four moments of returns for the seven assets over our full sample period, as well as three normality tests. The numbers in Table 1 were computed using monthly returns, but following Cumming et al (2014), their annualized values are shown in the table. There is significant negative skewness for all the asset classes except bonds, and significant positive kurtosis for every asset class. The normality tests confirm that all asset returns, except bonds, are non-normal at the 1% level on all three tests; and bond returns

[2] Agarawal et al (2013), Agarawal et al (2015), Aiken et al (2012), Fung and Hsieh (2000, 2002), Malkiel and Saha (2005) and Posthuma and Van der Sluis (2003).

are non-normal at the 5% level using the Jarque-Bera and Shapiro-Wilk tests. Therefore our data contains significant and widespread non-normality due to both skewness and kurtosis, making the use of mean-variance analysis questionable.

Asset class	1 Mean %	2 Std. Dev. %	3 Skewness	4 Kurtosis	Normality Tests		
					Jarque -Bera	Shapiro -Wilk	Anderson -Darling
Equities	9.813	14.881	-0.193^{***}	3.096^{***}	32.836^{***}	4.818^{***}	1.537^{***}
Bonds	5.442	3.613	-0.057	3.082^{**}	11.479^{**}	2.205^{**}	0.629
Real estate	13.116	20.268	-0.227^{***}	3.663^{***}	693.496^{**}	20.149^{**}	4.440^{***}
Commodities	2.623	22.037	-0.100^{**}	3.105^{***}	21.292^{***}	2.600^{***}	0.964^{***}
Private	10.731	24.203	-0.102^{**}	3.445^{***}	304.524^{**}	16.375^{**}	5.371^{***}
Emerging	7.020	23.195	-0.192^{***}	3.161^{***}	58.046^{***}	5.381^{***}	1.174^{***}
Hedge funds^{+}	4.979	8.243	-0.179^{***}	3.258^{***}	117.093^{**}	6.786^{***}	1.359^{***}

Table 1: The First Four Moments and Normality Tests, Annualized Returns - 1994-2015

$^{+}$ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The significance of departures from normality for skewness and kurtosis rely on D'Agostino et al (1990). Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns.

2d. Correlations. The correlations for monthly returns appear in Table 2. The highest correlation is 0.75, while 12 of the 30 correlations for alternative assets are less than 0.50, suggesting considerable diversification benefits are available from adding alternative assets to a portfolio of equities and bonds.

	Equities	Bonds	Real Estate	Commodities	Private Equity	Emerging Markets	Hedge Funds^{+}
Equities	*1*	-	-	-	-	-	-
Bonds	0.25	*1*	-	-	-	-	-
Real Estate	0.57	0.34	*1*	-	-	-	-
Commodities	0.26	0.12	0.17	*1*	-	-	-
Private Equity	0.75	0.23	0.61	0.38	*1*	-	-
Emerging Mkts.	0.73	0.25	0.48	0.37	0.73	*1*	-
Hedge Funds^{+}	0.63	0.31	0.32	0.38	0.62	0.74	*1*

Table 2: Correlations of Monthly Returns - 1994-2015

$^{+}$ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method.

3. Methodology

To allow for the non-stationarity of returns, our analysis is based on the 'rolling-windows' approach, with an estimation window of 36 (or 48) months used to compute the optimal asset allocations for each portfolio model over the next out-of-sample month. The estimation window is then advanced by one month, and the process repeated 228 (216) times.

3.1 The Portfolio Models. We use a wide variety of portfolio models to increase the generality of our results. Our 19 portfolio models can be divided into three main groups - mean-variance, *ad hoc* and utility functions. These are listed in Table 3, with further details of each model in the Appendix. Solving portfolio problems with a small number of assets in the presence of estimation errors can result in unrealistic asset weights, e.g. 100% of the assets in a single alternative asset; and upper bounds have been placed on the asset weights by many previous studies of alternative assets to rule out unrealistic solutions. Pension schemes have, on average, 25% of their assets in alternatives (Towers Watson, 2015), and this suggests that optimal solutions with more than half the assets in alternative assets, are unrealistic. So we impose an upper bound of 50% on the sum of the alternative asset allocations.

Mean Variance	1	Markowitz
	2	Markowitz with upper generalized constraints
	3	Markowitz with lower generalized constraints
	4	Bayes diffuse prior
	5	Bayes-Stein shrinkage
	6	Black-Litterman (*1/N*)
	7	Black-Litterman (minimum variance)
	8	Michaud - resampled efficient frontier
Ad hoc	9	Minimum variance
	10	Minimum variance with upper generalized constraints
	11	Minimum variance with lower generalized constraints
	12	*1/N*
	13	Combination - minimum variance & *1/N*
	14	Combination - minimum variance, *1/N* & Markowitz tangency
	15	Risk parity
	16	Reward to risk timing
Utility Function	17	CRRA utility (1st four moments)
	18	CRRA utility (1st four moments) with upper generalized constraints
	19	CRRA utility (1st four moments) with lower generalized constraints

Table 3: Portfolio Models

See the Appendix for details of these models.

3.2 Utility Function*.* Since our individual asset returns are non-normal, portfolio returns are also likely to be non-normal, and mean-variance analysis may be unsatisfactory because it considers only the first two moments of the return distribution. The usual solution is to maximise a utility function, and the two most widely used are the CRRA (or power) utility function, and the constant absolute risk aversion (CARA or exponential) utility function. Both of these utility functions imply investors have a positive preference for skewness (the third moment), and a negative preference for kurtosis (the fourth moment), as well as a positive preference for the mean (the first moment), and a negative preference for the variance (the second moment), Scott and Horvath (1980)[3]. Kallberg and Ziemba (1983) found that six utility functions, including CRRA and CARA, lead to similar optimal portfolios for horizon periods of up to one year. The CRRA utility function has the advantage that, since relative risk aversion is constant, the asset allocation is independent of investor wealth, allowing performance measurement to use either the level of wealth at the end of each period, or the return each period. So, in common with many previous researchers, we assume that investors have a time-separable CRRA utility function and unit initial wealth:-

$$U_t = (1+r_t)^{1-\lambda}/(1-\lambda) \tag{1}$$

where r_t is the portfolio return in period t, and λ is the investor's risk aversion parameter. Some researchers have maximised a utility function by using either numerical quadrature or Monte Carlo simulation. These approaches are only practical when there are just two or three assets, and we have seven assets, which makes these methods impractical. Therefore, in common with many previous studies, we use a Taylor series expansion[4]. A Taylor series expansion of a CRRA utility function only

[3] Table 1 documented the presence of negative skewness in asset returns which appeals to investors who exhibit imprudence; and excess positive kurtosis which is attractive to investors who display intemperance. So consideration of the third and fourth moments will tend to make these assets less attractive to investors with a CRRA utility function. (Eeckoudt and Schlesinger (2006).

[4] The Taylor series expansion is an approximation, and we use the first four terms to approximate the entire series. Garlappi and Skoulakis (2011) have found that, for the CRRA utility function and four asset portfolios, a Taylor series expansion of order four is reasonably accurate when portfolio returns are normal. When portfolio returns are non-normal its accuracy decreases as risk aversion increases, and for risk aversion of 10 it leads to a proportionate error of 3% in the CER.

converges for values of wealth between zero and twice the expected wealth, Loistl (1976). For our data the CRRA convergence limits are met.

4. Transactions Costs

Transactions costs are an important aspect of the asset allocation decision, particularly for alternative assets. This is the first study of two or more alternative assets to allow for transactions costs when computing the optimal portfolios, as well as when measuring performance. The value of the trades and the associated transactions costs involved in rebalancing a portfolio are affected by the initial asset allocation because a substantial alteration to the asset allocation generates higher transactions costs than one which is similar to the current allocation. This makes large rebalancings less attractive. Since the net returns from an asset allocation are affected by the previous asset allocation when transactions costs are included, the optimal asset allocations become path dependent. Even when the initial asset allocation is entirely in cash, if transactions costs differ between assets they will still affect the optimal solution.

We introduce transactions costs into our optimizing models by including the total transactions costs (TC_j) in the various objective functions:-

$$TC_j = \sum_{i=1}^{n} (| x_{ij} - x^*_{ij-1} |) T_i \qquad (2)$$

where T_i is the proportionate transactions cost for trading the i^{th} asset, and x^*_{ij-1} is the proportion of the value of the portfolio at the end of the previous period (i.e. $j-1$) in asset i, allowing for price changes during that period. We assume transactions costs are a linear function of trade value, and are the same for buys and sells[5]. In our mean-variance and utility models TC_j is subtracted from the expected portfolio return[6]. The effect of transactions costs on the asset allocation decision depends on the period over

[5] We only consider the variable transactions costs of trading assets, and ignore the fixed costs of establishing and maintaining the systems to invest in each asset. Inclusion of these costs in the models would reinforce the conclusion that investing in additional alternative assets is not beneficial.

[6] No transactions costs are included in the *ad hoc* models (except for combination: *1/N* & minimum variance and combination: *1/N*, minimum variance & Markowitz), although they are included when measuring performance.

which the transactions costs are expected to be spread, i.e. the expected holding period (Woodside-Oriakhi, et al, 2013). A survey of the equity trades of almost 1,000 active institutional managers for 2006-2009 by the IRRC Institute (2010) found the average expected holding period is 1.75 years (21 months), so we divided our transactions costs by 21 to get their expected effect on monthly returns[7].

The transactions costs for an institutional investor trading a particular asset vary over time as well as between investors making it hard to specify a single figure for an asset, and previous authors have used a very wide variety of transactions costs. For example, French (2008) estimated the transactions cost of trading US equities at 11 bps in 2006, while Kourtis (2015) used 100 bps. We estimate the proportionate transactions costs of trading equities in developed markets at 50 bps, as have DeMiguel et al (2009), Kirby and Ostdiek (2012) and Daskalaki and Skiadopoulos (2011). Since the returns for REITs and listed private equity are based on assets traded on developed equity markets we also use 50 bps for real estate, as have Clare et al (2016); and 50 bps for private equity. Emerging equity markets have higher transactions costs than developed equity markets, and we estimate these at 55 bps, as have Clare et al, (2016). Commodity investment is usually achieved via commodity futures which have a life of a few months before they are delivered, and to maintain a position for a year the futures position has to be rolled over several times. Clare et al (2016) use a transactions cost for commodities of 8 bps, and as we allow for rolling over the position four times per year, this gives an annual cost of about 32 bps. We adopt the estimate of 35 bps of Daskalaki and Skiadopoulos (2011) for commodities. Edwards et al (2007) have estimated the transactions cost of a $200,000 corporate bond trade at 17 bps, which is also the cost used by Clare et al (2016) for bond trades, and we also use this number.

The only unlisted asset class we consider is hedge funds. Investment in a fund of hedge funds is a cash inflow to the fund, which then invests in a range of hedge funds, without using organized financial markets. The management and performance fees for funds of hedge funds have been estimated at 1.78% per year by French (2008), and are deducted from their reported returns; so these costs are already allowed for in returns. Therefore the additional transactions costs paid by institutional

[7] The actual holding period found by the IRRC Institute is only 1.40 years.

investors when they invest in a fund of hedge funds are low, and following Hocht et al (2008), we use a transactions cost of 50 bps.

Since our portfolio models use monthly returns, before including the above transactions costs in our portfolio models we divide them by the expected holding period of 21 months, giving costs of 2.4 bps for equities, real estate, private equity, real estate and hedge funds, 2.6 bps for emerging markets, 1.7 bps for commodities and 0.8 bps for bonds. However, when measuring performance we include the full transactions cost of the trades involved in rebalancing the portfolio.

5. Markov Regime Switching

There is considerable evidence that asset returns, risks and correlations shift with the economic regime (e.g. Guidolin, 2011). In a study of US equities, Tu (2010) concludes that allowance for regime switching improves out-of-sample portfolio returns and should be employed. Our data covers three bull markets for US equities (October 1994 to January 2000, July 2002 to July 2007, and January 2009 onwards) separated by two major bear markets (January 2000 to July 2002 and July 2007 to January 2009). Therefore we allow the distributions of assets returns and the relationships between them to shift according to the prevailing economic regime, allowing us to study how the diversification benefits of alternative assets change with the economic regime. Only Sa-Aadu et al (2010) and Hocht et al (2008) have applied a Markov regime shifting model to portfolios with two or more alternative assets. They used the two-state Markov chain analysis of Hamilton (1989, 1990) which assumes that asset returns are a mixture of normals, and then uses maximum likelihood estimation. For portfolios with two alternative assets, the two-regime model of Sa-Aadu et al (2010) generated superior out-of-sample performance in terms of both Sharpe ratios and CERs using two, rather than one, regime.

Assuming the non-normality evidenced in Table 1 is due to asset returns being a mixture of two normals (one for each regime); and following Ang and Bekaert (2004) and Ang and Timmermann (2012) amongst others; we use MS_Regress (Perlin (2015) to fit a two state multi-variate regime-switching model to asset returns for each rolling window estimation period (36 or 48 months). For each month, when the smoothed conditional probability over the estimation period is higher (lower) than 50%, that month is classified as being in state 1 (state 2), (Kim, 1994). Based on the realisation of the regime in the last month of each estimation period (Ang and Bekaert, 2004), we compute the first four moments and the covariance matrix, as defined by Timmermann (2000) and Ang and Timmermann (2012), for the next out-of-sample month. We then use these estimates as inputs to our 19 portfolio models[8]. The results in Table 4 broadly support the mixture of two normals assumption. None of the skewness statistics, and only three of the kurtosis statistics (one in the low volatility regime, and two in the high volatility regime) are statistically significant at the 5% level. This contrasts with Table 1 where all but bonds exhibit skewness, and all the assets demonstrate kurtosis. So with a single regime all seven assets are highly non-normal, but with two regimes most assets become normal, and where they remain non-normal, the non-normality is much reduced.

	Low Volatility Regime				High Volatility Regime			
	Mean %	Std.Dev. %	Skewness	Kurtosis	Mean %	Std.Dev. %	Skewness	Kurtosis
Equities	15.929	10.703	−0.063	3.001	−4.252	21.165	−0.080	2.960
Bonds	4.090	3.496	−0.092*	3.103**	8.604	3.754	−0.035	3.049
Real estate	19.733	14.499	−0.093*	3.056*	−2.103	29.200	−0.129*	3.406***
Commodities	3.573	19.126	−0.063	3.041	0.437	27.726	−0.111	3.077
Private equity	23.228	14.618	−0.056	3.060*	−18.013	36.812	0.081	3.173***
Emerging	16.574	17.057	−0.012	2.993	−14.952	32.539	−0.105	3.023
Hedge funds+	7.109	5.745	−0.097*	2.991	0.080	12.117	−0.086	3.076

Table 4: The First Four Moments of the Two Regimes, Annualized Returns - 1994-2015

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The significance of departures from normality for skewness and kurtosis rely on D'Agostino et al (1990). Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns. The values for the first four moments have been annualized, Cumming et al (2014).

We now use the regime classification measure (RCM) of Ang and Bekaert (2002a) to investigate the success of our regime shifting analysis in distinguishing between the two regimes. The RCM statistic lies between zero and 100, where a score of 100 indicates the model cannot distinguish between the regimes. The RCM score for our two regimes is 12.855, and since this number is reasonably close to zero, it supports the view that we are distinguishing between two regimes.

[8] Note that the *1/N* model is regime independent.

Table 5 tests for significant differences between the means and variances of monthly returns for the two regimes across the sample period. Ang and Bekaert (2002c, 2004) find that for equities the high volatility regime has lower mean returns, while for bonds Ang and Bekaert (2002a, 2002b) find that the high volatility regime has higher returns. Table 5 shows that these conclusions also apply to our data. It shows that the low volatility regime has significantly higher mean returns for US equities, private equity and hedge funds, while bonds have significantly lower mean returns in the low volatility regime. We use the Levene (1960) test for unequal variances because it is robust to non-normality, and this shows that for every asset, except bonds, the variance of monthly returns in the high volatility regime is significantly higher than in the low volatility regime.

	Means[1]	Variances[2]
Equities	2.333**	62.625***
Bonds	−2.677***	0.397
Real estate	1.835	17.238***
Commodities	0.265	14.668***
Private equity	2.800***	48.161***
Emerging mkts.	2.368**	46.865***
Hedge funds[+]	1.415	44.396***

Table 5: Tests for an Equality of Means and Variances in the Two Regimes - 1994 to 2015
[+] Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. [1] t-test statistics for equal means with unequal variances. [2] Levene (1960) test statistics for unequal variances. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns.

To investigate the mixture of two normals assumption further, Table 6 has normality tests for the monthly returns in each of the two regimes we have identified. In every case, with the exception of bonds in the low volatility regime, all the normality test statistics for the two regimes are lower than in Table 1, and in some cases a lot lower. With two regimes equities, emerging markets and hedge fund returns are normal in both regimes on all three tests. In the high volatility regime bonds and commodities are normal according to all the tests, and private equity is normal at the 5% level in the low volatility regime on all three tests. So the use of two regimes has greatly increased the normality of returns of our seven assets, and most assets are now normally distributed.

	Low Volatility Regime			High Volatility Regime		
	Jarque -Bera	Shapiro -Wilk	Anderson -Darling	Jarque -Bera	Shapiro -Wilk	Anderson -Darling
Equities	1.452	1.075	0.749	1.903	1.404	0.429
Bonds	13.393**	2.290**	0.631	0.940	0.565	0.240
Real estate	6.071***	1.543*	0.574	70.062**	7.001**	2.323***
Commodities	2.958	2.177**	1.031***	3.966	1.453	0.375
Private equity	4.511*	1.170	0.384	12.686**	2.689**	0.944**
Emerging mkts.	0.146	1.269	0.480	1.807	1.283	0.277
Hedge funds+	3.554	1.387	0.443	3.172	1.216	0.329

Table 6: Normality Tests for Two Regimes for Monthly Returns - 1994-2015

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns.

6. Results

For each of our seven portfolios of assets and 19 portfolio models we generate a time series of 228 portfolio returns for three year rolling window estimation periods, and 216 returns for four year estimation periods. This produces (19×7) = 133 out-of-sample returns for each of our four cases - one regime and three years, one regime and four years, two regimes and three years, and two regimes and four years. We solve each case for three levels of lambda (2, 5 and 10), allowing an investigation of whether the benefits of diversification differ with risk aversion.

6.1. Performance Measurement. Smetters and Zhang (2014) show that, if portfolio returns are normally distributed, Sharpe ratios correctly rank portfolios; but if portfolio returns are non-normal, investor preferences may be required to rank portfolios. While the returns on our seven individual assets are clearly non-normal when we use one regime, it is possible that the out-of-sample returns on the portfolios formed using our 19 different models are normal[9]. In which case the use of Sharpe ratios is appropriate. We tested the normality of the 228 (216) out-of-sample returns for each of our 133 asset allocations for three (four) year estimation periods when λ = 5, and the results appear in Table 7. This

[9] The central limit theorem means portfolio arithmetic returns will tend to be more normal than individual asset returns.

shows that the vast majority of our 133 portfolios across the four cases have non-normal returns. Therefore the use of Sharpe ratios is problematic for our data, and we use CERs as our primary measure of portfolio performance. CERs are the riskless rate of return that gives the same utility as the risky returns on the portfolio. This certain equivalent return (CER) for a CRRA utility function is computed as:-

$$\text{CER} = (1-\lambda)\text{E}[\text{U}]^{1/(1-\lambda)} - 1 \tag{3}$$

where *E[U]* is the mean utility across our out-of-sample periods (Diris et al, 2015).

	Three Year Estimation Periods - 1997-2015					
Significance Level	One Regime			Two Regimes		
	Jarque -Bera	Shapiro -Wilk	Anderson -Darling	Jarque -Bera	Shapiro -Wilk	Anderson -Darling
Insignificant	0	0	0	0	0	0
10%	0	0	4	0	0	2
5%	0	0	10	2	0	11
1%	133	133	119	131	133	120
	Four Year Estimation Periods - 1998-2015					
Significance Level	One Regime			Two Regimes		
	Jarque -Bera	Shapiro -Wilk	Anderson -Darling	Jarque -Bera	Shapiro -Wilk	Anderson -Darling
Insignificant	0	0	0	0	0	6
10%	0	0	0	0	0	2
5%	0	0	8	4	0	12
1%	133	133	125	129	133	113

Table 7: Number of Portfolios with Non-Normal Out-of-Sample Returns when $\lambda = 5$, Three and Four Year Estimation Periods, One and Two Regimes

We regress these CER values on two sets of 0-1 dummies, where DA_i denotes the i^{th} asset set, and DT_j denotes the use of the j^{th} portfolio model.

$$CER_{ij} = \alpha + \sum_{i=2}^{7} \beta_i DA_i + \sum_{j=2}^{19} \delta_j DT_j + \varepsilon_{ij} \tag{4}$$

To allow the covariance matrix to be inverted, we include a constant term and drop the dummy variables DA_1, which indicates portfolios of equity and bonds, and DT_1 which indicates the Markowitz model[10]. Therefore the constant term in equation (4) is the predicted CER for an equity and bond portfolio formed using the Markowitz model, the β_i give the estimated difference between the CER for the i^{th} portfolio of assets and that for equities and bonds, and the δ_j give the estimated difference in the CER for the j^{th} model and that for Markowitz[11].

6.2 CER Results. The estimated coefficients for the DA_i for three different levels of risk aversion for our four combinations of regimes and estimation periods appear in Table 8. The coefficients of the six portfolios show the marginal effect of adding each alternative asset individually, or all five alternative assets together, to a portfolio of equities and bonds formed using the Markowitz model. The coefficients for the dummy variables of the other 18 portfolio models are not reported in Table 8 as a horse race between portfolio models is not the purpose of this research. The *DT* dummies are included in the regressions to control for the effects of using different portfolio models.

Across our four cases, portfolios containing all seven assets have significantly lower CERs for eleven regressions, while the twelfth coefficient for four years, one regime and λ =2 is negative but not significant. This indicates that portfolios of only equities and bonds are superior to those which also contain the five alternative assets. This result is generally supported by the results for adding individual alternative assets to the equities and bonds portfolio, although they are a bit weaker for two regimes. Out of sixty coefficients for the five individual alternative assets, four cases and three λ values; only nine are significantly positive, while 28 are significantly negative. This result indicates that diversification is harmful, rather than beneficial. This finding gets stronger as risk aversion increases. In section 7 we investigate the reasons for this unexpected result.

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[10] We have standardized the results from our 19 portfolio models to those of the Markowitz model because it is very widely used in the literature.

[11] We have not included any interactions between the DA_i and DT_j in equation (4) because there are (6×18) = 108 such interactions, and we only have one observation for each interaction.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	8.143***	6.545***	4.496***	5.768***	3.831***	1.779***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	−1.099*	−1.214**	−1.350**	-0.338	-0.384	-0.392
E&B & Hedge Funds+	−0.362*	−0.275	−0.278	0.210	0.319	0.433
E&B & Real Estate	0.183	−0.178	−0.711**	0.502**	0.035	-0.917**
E&B & Private Equity	−0.599*	−0.944**	−1.512**	0.512**	0.510**	0.354
E&B & Emerging Markets	0.246	0.109	−0.531**	1.670***	0.731***	0.247
All Seven Assets	−1.133*	−1.712**	−2.473**	-0.712***	-1.260***	-2.406**
18 model dummies, excluding Markowitz which is the comparator						
Adjusted R squared	0.731	0.621	0.736	0.508	0.538	0.683
	Four Year Estimation Periods - 1998-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	5.656***	4.677***	2.919***	4.921***	3.620***	1.767***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	-0.291	-0.748***	-1.141***	-1.577***	-2.247***	-2.448**
E&B & Hedge Funds+	-0.299	-0.317*	-0.246	-1.077***	-1.011***	-0.849**
E&B & Real Estate	0.160	-0.492***	-1.040***	0.555***	-0.254	-0.815**
E&B & Private Equity	-0.455**	-0.795***	-1.369***	0.534***	0.468**	0.023
E&B & Emerging Markets	0.886***	0.059	-0.705*	-0.251	-0.576***	-1.345**
All Seven Assets	-0.063	-1.014***	-2.430***	-0.415**	-0.996***	-2.161**
18 model dummies, excluding Markowitz which is the comparator						
Adjusted R squared	0.421	0.559	0.780	0.676	0.745	0.751

Table 8: Regression Co-efficients for Annualized CERs in Percent Using CRRA Utility Relative to Equities-Bonds and Markowitz - Three and Four Year Estimation Periods, and One and Two Regimes

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The dependent variables are the CERs. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns.

The predicted CERs from equity and bond portfolios formed using the Markowitz model (i.e. the constant terms in equation (4)) decrease as risk aversion increases. This is because although increasing the level of risk aversion when forming portfolios leads to an increase in the constant terms, this is more than outweighed by the decrease in the constant terms when higher risk aversion is used to evaluate these portfolios. So the net effect on the constant terms is a decline as risk aversion increases. The constant terms also decrease when two regimes and a four year estimation period are used, suggesting that the use of a single regime and three year estimation period is preferable.

6.3 *Sharpe Ratio Results*. As a robustness check, and to investigate the extent to which non-normal returns lead to different performance measurements from those provided by CERs, we also compute the Sharpe ratios for each of our 133 time series of out-of-sample returns. As for CERs, we regress the Sharpe ratios on the two sets of dummy variables in equation (4); and the results for our three levels of risk aversion and four cases appear in Table 9. Across our four cases, portfolios containing all seven assets have a significantly lower Sharpe ratio at the 1% level for all twelve regressions[12]. So, as for CERs, portfolios containing all seven assets are inferior to portfolios of just equities and bonds. Out of sixty coefficients for the five individual alternative assets, four cases and three λ values; only five are significantly positive, while 37 are significantly negative. The Sharpe ratios indicate even more strongly than the CER results that diversification is harmful. These results are consistent with the CER results, supporting the view that Sharpe ratios are a valid performance measure in the presence of non-normal portfolio returns.

The length of the estimation period and the number of regimes have a similar effect on the Sharpe ratio and CER constants. The constant terms in the Sharpe ratio regressions decrease when two regimes and four year estimation periods are used, suggesting that the use of a single regime and a three year estimation period is preferable. Unlike the CER regressions, the constant terms in the Sharpe ratio regressions increase with risk aversion. This is because the level of risk aversion affects the formation of portfolios, but not their evaluation using Sharpe ratios. As for CERs, higher risk aversion leads to higher Sharpe ratios, but this is not offset by higher risk aversion when evaluating these ratios.

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[12] The coefficients for the dummy variables of the other 18 portfolio models are not reported in Table 9. Conducting a horse races between portfolio models is not our purpose. Across our four cases, the coefficients for these models do not reveal any clear patterns of dominance or inferiority between portfolio models.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	0.673***	0.708***	0.745***	0.443***	0.446***	0.482***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	−0.180**	−0.172**	−0.168**	-0.070**	-0.074***	-0.078***
E&B & Hedge Funds^{+}	−0.048**	−0.037*	−0.040	0.049*	0.038	0.018
E&B & Real Estate	−0.049**	−0.049**	−0.052**	0.000	0.001	0.000
E&B & Private Equity	−0.133**	−0.132**	−0.127**	0.004	0.027	0.047*
E&B & Emerging Markets	−0.081**	−0.036	−0.020	0.092***	0.060**	0.080***
All Seven Assets	−0.217**	−0.224**	−0.220**	-0.134***	-0.135***	-0.148***
18 model dummies excluding Markowitz which is the comparator						
Adjusted R squared	0.848	0.832	0.843	0.769	0.782	0.785
	Four Year Estimation Periods - 1998-2015					
Regime	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	0.468***	0.547***	0.589***	0.402***	0.449***	0.473***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	-0.094***	-0.110***	-0.125***	-0.218***	-0.279***	-0.308***
E&B & Hedge Funds^{+}	-0.035*	-0.039*	-0.035**	-0.104***	-0.118***	-0.124***
E&B & Real Estate	-0.040**	-0.062***	-0.065***	0.013	-0.038	-0.052**
E&B & Private Equity	-0.107***	-0.101***	-0.089***	0.019	0.027	0.029
E&B & Emerging Markets	0.007	-0.010	-0.012	-0.078***	-0.075***	-0.070***
All Seven Assets	-0.107***	-0.133***	-0.153***	-0.118***	-0.117***	-0.125***
18 model dummies excluding Markowitz which is the comparator						
Adjusted R squared	0.889	0.884	0.886	0.851	0.864	0.870

Table 9: Regression Co-efficients for Annualized Sharpe Ratios Relative to Equities-Bonds and Markowitz - Three and Four Year Estimation Periods, and One and Two Regimes

$^{+}$ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The dependent variables are the Sharpe Ratios. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns.

These results for both CERs and Sharpe ratios indicate that adding alternative assets to a portfolio of equities and bonds tends to lead to a deterioration in out-of-sample performance. This is particularly apparent when all five alternative assets are included to the portfolio. Therefore, in contradiction to Markowitz, diversification is harmful.

7. Explanations for Harmful Diversification

Markowitz (1952) has shown that diversification improves the performance of mean-variance portfolios, assuming no estimation errors or transactions costs, correlations between assets are less than unity, and returns are normally distributed or utility is quadratic[13]. There is also a substantial literature which has demonstrated empirically that diversification reduces portfolio risk, e.g. Evans and Archer, (1968). However these empirical demonstrations of the benefits of diversification use a *1/N* asset allocation strategy which takes no account of estimation errors, transactions costs, the correlations between assets, the distribution of returns and the investor's utility function when forming portfolios. Since our correlations are well below unity, this suggests harmful diversification is caused by estimation errors, transactions costs, non-normality, or some mixture of these three explanations. Therefore we now investigate the causes of harmful diversification by exploring the effects of transactions costs, non-normality and estimation errors.

7.1 Transactions Costs. If all assets have identical variable transactions costs and there are no fixed costs for investing in an additional asset, the asset allocation and relative performance of portfolios containing different numbers of assets are unaffected when allowance is made for transactions costs. However, differential variable and fixed transactions costs exist, and so transactions costs can affect portfolio composition and relative portfolio performance. Jennings and Payne (2016) argue that alternative assets have such high transactions costs that they make diversification into these assets unattractive. In section 4 our estimates show that emerging markets have the highest transactions costs; equities, commodities, hedge funds, real estate and private equity all have the same transactions costs; and bonds have markedly lower transactions costs than the other assets. Therefore any transactions cost effect on performance is not due to alternative assets having markedly higher

[13] If one asset has a very high return and low risk, mean-variance analysis can lead to an undiversified portfolio, but in competitive markets this would be an exceptional situation.

transactions costs than equities and bonds, but to the increased turnover generated by rebalancing the portfolio across more assets, and to the choice of different asset weights due to the incorporation of transactions costs in the objective function.

The average transactions costs across our 19 portfolio models appear in Table 10. These numbers represent the decrease in expected returns due to transactions costs. The transactions costs for two regimes are markedly higher than for one regime, as the use of two regimes leads to much more trading. The average transactions costs for seven asset portfolios are the highest, and those for two asset portfolios the lowest, indicating that transactions costs reduce the expected returns of seven asset portfolios by more than they reduce those of two asset portfolios. This differential effect on average expected returns is about 0.34%, and suggests that the presence of transactions costs might be partly responsible for harmful diversification. But this ignores the possibility that the inclusion of transactions costs leads to the choice of different asset weights, which then affect the portfolio's first four moments. Therefore we need to compare the performance of portfolios with and without transactions costs.

To investigate the effects of transactions costs on portfolio performance we re-estimated our 19 models with zero transactions costs when both forming and evaluating portfolios, and the results for $\lambda = 5$ for our four cases appear in Table 11. Comparing these results with those in Tables 8 and 9, the constant terms are increased for both the CERs and Sharpe ratios, partly due to the absence of transactions costs; while the smaller negative slope coefficients indicate that the harmful diversification effect is reduced, but still clearly present. So the presence of transactions costs does not explain our finding that diversification is harmful; they just cause a modest increase in the magnitude of the harm.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Equities and Bonds (E&B)	0.1786	0.190	0.1900	0.7874	0.7030	0.7383
E&B & Commodities	0.2778	0.279	0.2563	1.0828	0.9751	0.9636
E&B & Hedge Funds+	0.2584	0.272	0.2807	0.9932	0.9206	0.9172
E&B & Real Estate	0.3069	0.327	0.3114	0.9047	0.8163	0.8292
E&B & Private Equity	0.2920	0.295	0.2779	1.0042	0.9533	0.9570
E&B & Emerging Markets	0.2753	0.257	0.2481	1.0386	0.9552	0.8964
All Seven Assets	0.5353	0.505	0.4638	1.1427	1.0787	1.1100
	Four Year Estimation Periods - 1998-2015					
Regime	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Equities and Bonds (E&B)	0.1788	0.1684	0.1721	0.7936	0.7528	0.7082
E&B & Commodities	0.2637	0.2400	0.2284	0.9781	0.9839	0.9089
E&B & Hedge Funds+	0.2587	0.2385	0.2429	0.9971	1.0031	0.9668
E&B & Real Estate	0.2890	0.2715	0.2685	0.8788	0.8828	0.8313
E&B & Private Equity	0.2849	0.2471	0.2318	1.0239	0.9397	0.8465
E&B & Emerging Markets	0.2548	0.2200	0.2162	1.0209	0.9457	0.8262
All Seven Assets	0.5096	0.4557	0.4152	1.2368	1.1375	1.0223

Table 10: Average Reduction in Percentage Annualized Returns Due to Transactions Costs for the 19 Portfolio Models - 3 Year Estimation Period

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method.

	CERs			
Rolling Window	Three Years		Four Years	
Regime	One Regime	Two Regimes	One Regime	Two Regimes
Constant	6.800***	5.285***	5.001***	5.101***
Equities and Bonds	Comparator			
E&B & Commodities	−1.100***	-0.064	-0.617***	-2.071***
E&B & Hedge Funds+	−0.154	0.538**	-0.215	-0.738***
E&B & Real Estate	−0.069	0.213	-0.413**	-0.174
E&B & Private Equity	−0.800***	0.905***	-0.651***	0.681***
E&B & Emerging	0.204	1.144***	0.124	-0.404**
All Seven Assets	−1.413***	-0.895***	-0.743***	-0.661***
18 model dummies excluding Markowitz which is the				
Adjusted R squared	0.648	0.534	0.500	0.707
	Sharpe Ratios			
Rolling Window	Three Years		Four Years	
Regimes	One Regime	Two Regimes	One Regime	Two Regimes
Constant	0.730***	0.575***	0.578***	0.591***
Equities and Bonds	Comparator			
E&B & Commodities	−0.161***	-0.036	-0.099***	-0.265***
E&B & Hedge Funds+	−0.022	0.079***	-0.025	-0.073***
E&B & Real Estate	−0.042**	0.006	-0.058***	0.043*
E&B & Private Equity	−0.119***	0.062**	-0.093***	0.041*
E&B & Emerging	−0.031	0.090***	-0.010	-0.073***
All Seven Assets	−0.198***	-0.115***	-0.115***	-0.112***
18 model dummies excluding Markowitz which is the				
Adjusted R squared	0.838	0.767	0.885	0.840

Table 11: Regression Co-efficients for Annualized CERs in Percent and Sharpe Ratios Relative to Equities-Bonds and Markowitz λ = 5 Without Transactions Costs - Three and Four Year Estimation Periods, One and Two Regimes

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The dependent variables are CERs in the top panel, and Sharpe ratios in the bottom panel. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns.

7.2 Non-Normal Returns. Tu and Zhou (2004) show that non-normal asset returns lead to substantial changes in portfolio weights, but have only a small effect on quadratic utility. Jondeau and Rockinger (2006) have found that the mean-variance portfolio is a good approximation to that which maximises expected CRRA and CARA utility for moderate departures from normality. But for large departures, using the first four moments may be superior to mean-variance in terms of the expected return. Other researchers have also found that the performance of mean-variance analysis suffers when returns are non-normal, see Cvitanic et al (2008), Cremers et al (2004), Cumming et al (2014) and Xiong and Idzorek (2011). These results suggest that, when the departure from normality is substantial and volatility is high, there can be important differences between the performance of mean-variance portfolios and those which also rely on higher moments.

In Table 1 individual asset returns are shown to be non-normal, while Table 5 indicates that the use of two regimes leads to a big reduction in this non-normality. For both one and two regimes Table 7 reveals that portfolio returns are highly non-normal. This suggests that portfolio models which allow for non-normality will produce higher CERs than models which do not. To test this we compare the performance of the three models which maximise CRRA utility (which includes the first four moments); with that of the corresponding three Markowitz models (which only include the first two moments). For each of our four cases we rerun the regression in equation (4), except that we vary the benchmark portfolio. We used the three Markowitz models (1, 2 & 3 in Table 3) in turn as the benchmark portfolio model, in conjunction with equities and bonds. The estimated coefficients for the CRRA model (17, 18 & 19 in Table 3) corresponding to each of the three benchmark Markowitz portfolio models appear in Table 12, together with their significance levels in brackets. The coefficients in this table are very small, and a very long way from being statistically significant, indicating that allowing for the non-normality of asset returns has no discernable effect on portfolio performance.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
CRRA relative to Markowitz	0.004 (0.990)	0.083 (0.811)	−0.055 (0.901)	0.003 (0.995)	0.000 (1.000)	−0.010 (0.987)
CRRA relative to Markowitz, both with upper generalised constraints	−0.012 (0.972)	−0.001 (0.998)	−0.071 (0.873)	−0.001 (0.997)	−0.017 (0.965)	−0.036 (0.952)
CRRA relative to Markowitz, both with lower generalized constraints	0.000 (0.999)	0.043 (0.902)	−0.065 (0.884)	0.004 (0.991)	−0.011 (0.978)	−0.029 (0.962)
	Four Year Estimation Periods - 1998-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
CRRA relative to Markowitz	0.002 (0.994)	0.052 (0.864)	0.018 (0.967)	0.001 (0.998)	0.002 (0.995)	0.018 (0.973)
CRRA relative to Markowitz, both with upper generalised constraints	0.015 (0.962)	0.029 (0.925)	−0.027 (0.952)	−0.005 (0.986)	−0.008 (0.981)	0.006 (0.990)
CRRA relative to Markowitz, both with lower generalized constraints	0.007 (0.983)	0.066 (0.829)	0.026 (0.954)	−0.002 (0.994)	−0.007 (0.984)	0.000 (1.000)

Table 12: Annualized Percentage Changes in CERs when the Portfolio Model Allows for the Non-Normality of Asset Returns

Significance levels appear in brackets. The dependent variables are CERs for the three Markowitz models (1, 2 & 3 in Table 3).

We repeat the analysis in Table 12, but for Sharpe ratios, and the results appear in Table 13. The coefficients are even smaller than for CERs, again indicating no effect for non-normality.

When a quadratic utility function is used to measure performance the mean-variance models do not require normality. So we re-evaluate the performance of our 19 models using a quadratic, rather than a CRRA utility function, in the presence of transactions costs and estimation errors. The patterns of significant results in Table 14 for CERs based on a quadratic utility function are very similar to those in Tables 8 and 12 for a CRRA utility function, indicating that the normality assumption does not affect our conclusions. These results for a quadratic utility function are also consistent with the finding in Tables 9 and 13 that harmful diversification remains present when the Sharpe ratio, which assumes normality, is used to measure performance.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
CRRA relative to Markowitz	0.000 (0.993)	0.008 (0.834)	−0.011 (0.765)	0.000 (0.997)	0.000 1.000)	0.000 (0.992)
CRRA relative to Markowitz, both with upper generalised constraints	−0.001 (0.987)	0.000 (0.996)	−0.011 (0.763)	0.000 (0.996)	−0.001 (0.977)	−0.002 (0.959)
CRRA relative to Markowitz, both with lower generalized constraints	0.000 (0.999)	0.004 (0.916)	−0.012 (0.728)	0.000 (0.994)	−0.001 (0.983)	−0.002 (0.969)
	Four Year Estimation Periods - 1998-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
CRRA relative to Markowitz	0.001 (0.985)	0.004 (0.897)	−0.013 (0.674)	0.000 (1.000)	0.000 (0.995)	0.001 (0.987)
CRRA relative to Markowitz, both with upper generalised constraints	0.002 (0.963)	0.002 (0.940)	−0.010 (0.758)	−0.001 (0.989)	0.000 (0.991)	0.000 (0.998)
CRRA relative to Markowitz, both with lower generalized constraints	0.001 (0.978)	0.005 (0.865)	−0.008 (0.784)	0.000 (0.993)	−0.001 (0.989)	0.000 (0.998)

Table 13: Changes in Annualized Sharpe Ratios when the Portfolio Model Allows for the Non-Normality of Asset Returns

Significance levels appear in brackets. The dependent variables are Sharpe ratios for the three Markowitz models (1, 2 & 3 in Table 3).

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	7.874***	6.434***	4.759***	5.647***	3.919***	2.249***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	-1.037***	-1.144***	-1.243***	-0.330	-0.409*	-0.473*
E&B & Hedge Funds+	-0.341*	-0.256	-0.247	0.190	0.259	0.314
E&B & Real Estate	0.191	-0.104	-0.471**	0.498**	0.159	-0.316
E&B & Private Equity	-0.552***	-0.850***	-1.264***	0.484**	0.441**	0.166
E&B & Emerging	0.247	0.165	-0.309	1.574***	0.719***	0.344
All Seven Assets	-1.044***	-1.529***	-2.008***	-0.647***	-1.110***	-1.855***
18 model dummies, excluding Markowitz which is the comparator						
Adjusted R squared	0.734	0.621	0.725	0.512	0.528	0.705

	Four Year Estimation Periods - 1998-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	5.536***	4.688***	3.314***	4.828***	3.651***	2.091***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	-0.262	-0.684***	-1.023***	-1.497***	-2.134***	-2.315***
E&B & Hedge Funds+	-0.284	-0.298*	-0.232	-1.029***	-0.961***	-0.801***
E&B & Real Estate	0.182	-0.315*	-0.707***	0.536***	-0.194	-0.636*
E&B & Private Equity	-0.414**	-0.714***	-1.171***	0.508***	0.448**	0.032
E&B & Emerging	0.859***	0.174	-0.431**	-0.215	-0.413**	-0.837***
All Seven Assets	-0.036	-0.826***	-1.794***	-0.383**	-0.871***	-1.745***
18 model dummies, excluding Markowitz which is the comparator						
Adjusted R squared	0.425	0.526	0.768	0.678	0.750	0.772

Table 14: Regression Co-efficients for Annualized Percentage CERs Using Quadratic Utility Relative to Equities-Bonds and Markowitz - Three and Four Year Estimation Periods, and One and Two Regimes

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The dependent variables are the CERs computed using quadratic utility. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns.

7.3 Estimation Errors. In practice portfolios are generally formed using some form of mean-variance model, which introduces estimation errors into the optimization process. Chopra and Ziemba (1993), Kallberg and Ziemba (1984), Kan and Zhou (2007), Bengtsson (2004), Cho (2011) and Palczewski and

Palczewski (2014) have shown that errors in expected returns have a bigger effect on portfolio performance than do errors in the variances and covariances[14]. Therefore we concentrate on the effects of errors in estimating expected returns. The mean-variance portfolio model tends to over (under) weight assets whose expected returns are over (under) estimated, or whose variance and covariances are under (over) estimated, Michaud and Michaud (2008). This has two implications. First, there is a positive correlation between estimation errors and portfolio weights. Second, with seven assets in the portfolio the effects of estimation errors are bigger than when only two or three assets available[15].

Errors in estimating out-of-sample asset returns may be higher for alternative assets than for equities and bonds, making their inclusion less attractive. In Table 15 we measure estimation errors from the historical means in three ways - mean absolute error (MAE), mean squared error (MSE) and their standard deviation (SD)[16]. This shows that bonds clearly have the lowest estimation errors for out-of-sample returns. Hedge funds are the next most predictable asset, followed by equities. The other four alternative assets all have higher estimation errors than equities and bonds, suggesting that the benefits of diversification into alternatives may be reduced by their higher estimation errors.

To explore the first implication of Michaud and Michaud (2008) we compute the correlation between estimation errors (i.e. the expected asset return less the actual out-of-sample asset return) for our seven assets and the asset weights (w_{ijk}) for each of our 19 portfolio models, for i = 1 to 7 assets, k = 1 to 7 portfolios using j = 1 to 228 (216) observations. This produces 24 correlations for each portfolio model, value of λ, estimation period and number of regimes. Table 16 has the percentage of

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[14] We are concerned with the effects of estimation errors on portfolio performance, not asset weights.

[15] Ledoit and Wolf (2004) show that if asset returns are independently and identically distributed, the higher is the ratio N/T, where N is the number of assets and T is the number of observations, the greater are the errors in covariance matrices estimated using historic returns. Since $\delta(N/T)/\delta N > 0$ an increase in the number of assets, with an unchanged number of observations, increases the errors in estimating the covariance matrix of the resulting portfolio. As the ratio N/T is a linear function of N, portfolio problems with seven assets and 36 (48) observations have an N/T ratio that is 3.5 (3.5) times larger than for portfolios with only two assets.

[16] Welch and Goyal (2008) and Bossaerts and Hillion (1999) have shown the historic mean is the best way to estimate the expected equity return, and Thornton and Valente (2012) have reached a similar conclusion for expected bond returns.

these 24×19 = 456 correlations that are significantly different from zero at the 10% level[17] for each combination of λ, estimation period and number of regimes. These results support a positive link between estimation errors and asset weights for all four of our cases, indicating that errors in estimating returns affect portfolio allocations; with an over (under) estimate of an asset's returns leading to a increase (decrease) in its portfolio weight.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
	MAE	MSE	SD	MAE	MSE	SD
Equities	41.92%	29.68%	15.76%	42.57%	50.79%	15.87%
Bonds	9.20%	1.42%	3.44%	9.50%	22.44%	3.53%
Commodities	63.22%	65.63%	23.39%	62.37%	62.74%	22.51%
Hedge Funds⁺	22.30%	8.86%	8.61%	23.21%	9.51%	8.83%
Real estate	52.44%	57.12%	21.86%	54.50%	61.32%	22.41%
Private Equity	63.26%	83.53%	26.44%	64.54%	84.26%	26.23%
Emerging Markets	63.67%	71.40%	24.45%	64.43%	75.41%	24.80%
	Four Year Estimation Periods - 1998-2015					
Regimes	One Regime			Two Regimes		
	MAE	MSE	SD	MAE	MSE	SD
Equities	41.48%	29.55%	15.72%	42.39%	56.05%	16.21%
Bonds	9.24%	1.43%	3.45%	9.59%	23.26%	3.57%
Commodities	63.36%	66.25%	23.52%	64.22%	70.11%	23.85%
Hedge Funds⁺	21.44%	8.40%	8.38%	22.07%	8.84%	8.55%
Real estate	53.32%	58.64%	22.15%	55.30%	66.51%	23.39%
Private Equity	64.42%	84.86%	26.65%	65.99%	94.24%	27.82%
Emerging Markets	63.64%	70.65%	24.32%	66.06%	78.70%	25.45%

Table 15: Annualized Estimation Errors for Historic Estimates of Monthly Percentage Returns - Three and Four Years and One and Two Regimes

⁺ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. This table shows annualized returns. MAE is mean of the annualized absolute errors, MSE is mean of the squared annualized errors, and SD is standard deviation of the estimation errors. The figures for two regimes are averages across the portfolios in which an asset appears.

[17] The significance of these correlations from zero relies on a *t*-test ($t = \rho\sqrt{[(n-2)(1-\rho^2)]}$) which holds approximately if the value of the underlying variables are non-normal, Stuart et al (1999, sections 27.20 and 27.23).

Estimation Period	λ	One Regime		Two Regimes	
		Positive	Negative	Positive	Negative
Three Years	2	25.4%	0	30.3%	0
	5	23.9%	0	30.3%	0
	10	20.2%	0	26.5%	0
Four Years	2	32.2%	0	36.8%	0
	5	28.5%	0	36.6%	0
	10	22.8%	0	34.6%	0

Table 16: Percentage of the Correlations Between Estimation Errors and Asset Weights Across All 19 Models that are Significantly Different from Zero at the 10% Level, Three and Four Years and One and Two Regimes

The second prediction of Michaud and Michaud (2008) is that portfolio performance is degraded by the inclusion of additional assets. Kan and Zhou (2007) have shown that, if asset returns are multivariate normal and independently and identically distributed, Markowitz portfolios formed with short sales permitted and using historic returns and covariances have a worse out-of-sample performance than portfolios formed using the population values. If the covariance matrix is known and estimation errors are confined to expected returns, the performance loss from using historic returns simplifies to $N/(2\lambda T)$, where N is the number of assets in the portfolio, T is the number of observations, and λ is the investor's risk aversion coefficient. This shows that increasing the number of assets in a portfolio increases the loss of performance, as does a decrease in risk aversion (Kan and Zhou, 2007). This finding is consistent with our empirical finding that portfolios with more assets have a worse performance[18].

To quantify the effects of estimation errors on portfolio returns we compute the mean squared errors (MSE) (expected return minus actual return) for every out-of-sample period for each of our seven portfolios. These errors incorporate the efforts of various portfolio models to counteract the effects of estimation errors, as well as the error amplification of mean-variance models. Table 17 has the MSE

[18] In addition, even when there are no estimation errors, if asset returns are highly non-normal the introduction of additional assets in a portfolio can increase portfolio risk. Los (2003) shows that, if the stability exponent (which determines the kurtosis of the distribution of returns) is less than unity, *1/N* diversification can increase portfolio risk.

averaged across the 19 portfolio models. For each value of λ the portfolios with equities, bonds and hedge funds have slightly lower MSE values than portfolios with just equities and bonds, and the portfolios with all seven assets have the highest MSE values. The MSE values drop as λ increases, indicating that the effects of estimation errors are a smaller problem for risk averse investors[19]. These results are consistent with the theoretical predictions of Kan and Zhou (2007) that portfolios with more assets and those formed by less risk averse investors have higher estimation errors, leading to worse portfolio performance.

Estimation Period	Portfolios	One Regime			Two Regimes		
		2	5	10	2	5	10
Three Years	Equities and Bonds (E&B)	8.08	7.00	5.65	9.56	8.59	7.32
	E&B & Commodities	10.56	8.74	6.86	10.29	8.90	7.48
	E&B & Hedge Funds+	8.05	6.93	5.56	8.16	7.08	6.04
	E&B & Real Estate	11.18	9.59	7.79	13.15	11.66	9.98
	E&B & Private Equity	12.24	10.60	8.52	11.72	9.91	8.67
	E&B & Emerging Markets	13.20	11.02	8.81	14.85	12.70	10.83
	All Seven Assets	13.72	11.91	9.43	14.55	12.91	11.01
Four Years	Equities and Bonds (E&B)	7.67	6.51	4.75	9.79	8.45	7.34
	E&B & Commodities	11.19	8.96	5.95	12.49	10.11	8.02
	E&B & Hedge Funds+	7.57	6.40	4.62	8.11	6.97	5.73
	E&B & Real Estate	11.43	10.01	6.97	12.01	10.31	8.74
	E&B & Private Equity	13.02	10.81	7.56	10.72	9.32	8.47
	E&B & Emerging Markets	12.76	10.72	7.42	12.65	11.03	9.49
	All Seven Assets	14.39	12.25	8.80	15.23	13.08	11.34

Table 17: Average Across the 19 Models of the Annualized Mean Squared Errors for Expected Compared to Actual Portfolio Percentage Returns for λ = 2, 5 and 10, Three and Four Years, and One and Two Regimes

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method.

7.3.1 Estimation Errors and Diversification. Section 7.3 found that estimation errors are larger for alternative assets, and that these estimation errors have a positive correlation with portfolio asset

[19] The MAEs and standard deviations give broadly similar rankings - the equities and bonds have the lowest values, and the portfolios of all seven assets have the highest values.

weights. We also demonstrated that portfolios with alternative assets have larger differences between expected and actual portfolio returns than equity-bond portfolios. These results are consistent with the conclusion that alternative assets have larger estimation errors, which adversely affect portfolio asset weights, leading to worse out-of-sample performance for portfolios with alternative assets. To investigate further we re-solve our 19 models with the estimation errors removed to see if this is responsible for harmful diversification.

We approximate the removal of estimation errors by setting the out-of-sample expected returns each period to their actual values, and estimate the second, third and fourth moments and covariance matrix once using data for the entire sample period (1994-2015). With no estimation errors the Bayes diffuse prior and the Bayes-Stein shrinkage models converge to the Markowitz model, while *1/N* is unaffected by estimation errors. In the absence of estimation errors, but with transactions costs and non-normal returns, Table 18 shows that in terms of both CERs and Sharpe ratios, three asset portfolios perform better than two asset portfolios, and seven asset portfolios have a much better performance than two asset portfolios[20]. The significant positive coefficients, particularly for the seven asset portfolios, indicate that even when transactions costs and non-normality are present, diversification is highly beneficial if there are no estimation errors. This implies that harmful diversification is primarily due to estimation errors, not transactions costs or non-normality.

[20] Table 18 does not contain any results for two regimes as, in the absence of estimation errors, returns are the same in both regimes.

	Three Year Estimation Periods - 1997-2015					
Regimes	CERs			Sharpe Ratios		
Risk Aversion	2	5	10	2	5	10
Constant	30.193***	28.752***	26.143***	2.765***	2.752***	2.694***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	7.154***	6.745***	5.886***	0.426***	0.420***	0.400***
E&B & Hedge Funds^{+}	0.206	0.279	0.406	0.052	0.055	0.066
E&B & Real Estate	5.628***	5.116***	4.162***	0.197**	0.195**	0.174**
E&B & Private Equity	5.756***	4.651***	3.141**	-0.016	−0.029	-0.054
E&B & Emerging Markets	6.157***	5.345***	4.031***	0.179**	0.176**	0.163*
All Seven Assets	15.000***	13.828***	11.826***	0.687***	0.676***	0.642***
18 model dummies excluding Markowitz which is the comparator						
Adjusted R squared	0.892	0.886	0.875	0.922	0.919	0.916
	Four Year Estimation Periods - 1998-2015					
Regime	CERs			Sharpe Ratios		
Risk Aversion	2	5	10	2	5	10
Constant	29.338***	27.974***	25.483***	2.734***	2.723***	2.667***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	7.482***	7.035***	6.113***	0.431***	0.427***	0.408***
E&B & Hedge Funds^{+}	0.245	0.318***	0.441	0.062	0.065	0.076
E&B & Real Estate	5.791***	5.250***	4.258***	0.198**	0.196**	0.175**
E&B & Private Equity	6.106***	4.927***	3.322**	−0.013	−0.026	−0.051
E&B & Emerging Markets	6.503***	5.656***	4.281***	0.191**	0.189**	0.175**
All Seven Assets	15.622***	14.377***	12.259***	0.701***	0.692***	0.658***
18 model dummies excluding Markowitz which is the comparator						
Adjusted R squared	0.889	0.882	0.872	0.925	0.922	0.918

Table 18: Regression Coefficients for Annualized CERs in Percent and Sharpe Ratios Relative to Equities-Bonds and Markowitz with No Estimation Errors - Three and Four Year Estimation Periods and One Regime

$^{+}$ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The dependent variables are CERs in the left hand panels and Sharpe ratios in the right hand panels. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns.

7.3.2 Estimation Errors and the Credit Crisis. Our conclusions may be influenced by unusual events such as the credit crisis of 2007-2009 which falls within our data set. For example, Huang and Zhong (2013) found that the benefits of diversification into alternative assets decreased during the credit crisis, Chan et al (2011) conclude that the benefits of diversification are lower during crisis periods, and Arouri et al (2014) report a decrease in diversification benefits during contagion periods. So we repeat the analysis in Table 15 separately for the credit crisis[21], and non-credit crisis periods. The increases in the annualized MAE, MSE and SD percentages during the credit crisis, relative to the non-crisis, appear in Table 19. This table shows that estimation errors increased substantially during the credit crisis. Since we have previously shown that harmful diversification is caused by estimation errors, the exclusion of the credit crisis will weaken our finding of harmful diversification. Table 19 indicates that the increase in estimation errors during the credit crisis is smallest for bonds, followed by hedge funds, and then equities and commodities. Emerging markets, private equity and real estate have larger increases than bonds, hedge funds, equities and commodities. Therefore we expect the impact of excluding the credit crisis on harmful diversification to be largest for emerging markets, private equity and real estate.

[21] Following Iyer et al (2014) and Gorton (2009), we define the period of the credit crisis as the 20 months of August 2007 to March 2009.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
	MAE	MSE	SD	MAE	MSE	SD
Equities	20.29%	36.81%	6.37%	18.15%	83.39%	7.72%
Bonds	4.55%	1.71%	1.83%	5.50%	25.59%	2.32%
Commodities	48.24%	106.55%	14.85%	31.09%	57.33%	8.90%
Hedge Funds⁺	17.62%	14.96%	5.01%	20.92%	16.06%	6.16%
Real estate	58.64%	183.00%	23.09%	59.56%	170.98%	23.36%
Private Equity	51.07%	165.04%	14.68%	45.21%	120.08%	14.36%
Emerging Markets	53.07%	118.77%	14.03%	47.07%	112.36%	15.45%
	Four Year Estimation Periods - 1998-2015					
Regimes	One Regime			Two Regimes		
	MAE	MSE	SD	MAE	MSE	SD
Equities	21.15%	38.49%	6.53%	19.67%	107.80%	8.88%
Bonds	4.40%	1.71%	1.83%	5.52%	31.32%	2.31%
Commodities	50.06%	111.66%	15.09%	44.74%	102.18%	14.01%
Hedge Funds⁺	19.01%	15.77%	5.36%	19.36%	13.81%	5.51%
Real estate	58.77%	188.65%	23.15%	64.61%	201.70%	25.95%
Private Equity	52.96%	175.97%	15.12%	51.10%	161.34%	18.76%
Emerging Markets	54.53%	123.75%	14.54%	55.20%	145.77%	19.13%

Table 19: Differences in the Annualized Estimation Errors for Historic Estimates of Monthly Percentage Returns: Credit Crisis Period Minus the Non-Credit Crisis Period - Three and Four Years and One and Two Regimes

⁺ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. This table shows annualized returns. MAE is mean of the annualized absolute errors, MSE is mean of the squared annualized errors, and SD is standard deviation of the estimation errors. The figures for two regimes are averages across the portfolios in which an asset appears.

To investigate this we exclude the utility values for the credit crisis and recompute Table 8, with the results in Table 20. These results are substantially different from those in Table 8. Diversification into real estate, private equity and emerging markets is now beneficial, although diversification into commodities remains harmful. There is also some limited evidence that diversification into hedge funds remains harmful when the credit crisis is omitted. Diversification into all of the five alternative assets is mostly beneficial when the credit crisis is excluded, although for highly risk averse investors there is still some evidence of harmful diversification. These results are consistent with those in Table 19, as the assets with the largest increase in estimation errors during the credit crisis are also those where diversification is harmful for the full sample, but beneficial outside the credit crisis period. This supports the conclusion that the harmful diversification results for real estate, private equity and emerging markets are due to the large estimation errors for these assets during the credit crisis. However diversification into commodities, and to a lesser extent hedge funds; with their smaller estimation errors during the credit crisis, remain harmful when the credit crisis is removed.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	9.902***	8.096***	5.480***	7.578***	5.633***	3.519***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	-1.240***	-1.210***	-1.132***	-0.858***	-0.846***	-0.755***
E&B & Hedge Funds^{+}	-0.258	-0.215	-0.208	-0.014	0.005	-0.021
E&B & Real Estate	0.924***	0.482*	0.060	1.934***	1.414***	0.777***
E&B & Private Equity	0.327	0.027	-0.393	1.617***	1.610***	1.566***
E&B & Emerging Markets	1.329***	1.058***	0.418*	2.085***	1.249***	0.745***
All Seven Assets	0.274	-0.303	-0.835***	0.680***	0.081	-0.529**
18 model dummies, excluding Markowitz which is the comparator						
Adjusted R squared	0.825	0.653	0.380	0.764	0.680	0.613
	Four Year Estimation Periods - 1998-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	7.588***	6.367***	4.233***	6.027***	4.612***	2.503***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	-0.112	-0.556**	-0.819***	-1.003***	-1.813***	-2.011***
E&B & Hedge Funds^{+}	-0.249	-0.247	-0.183	-0.915***	-0.917***	-0.776**
E&B & Real Estate	0.905***	0.509**	-0.026	1.679***	0.761***	0.256
E&B & Private Equity	1.039***	0.611***	0.121	1.777***	1.826***	1.726***
E&B & Emerging Markets	2.033***	1.377***	0.677***	0.223	-0.019	-0.614**
All Seven Assets	2.225***	0.588***	-0.205	1.451***	1.023***	0.193
18 model dummies, excluding Markowitz which is the comparator						
Adjusted R squared	0.743	0.606	0.511	0.810	0.812	0.726

Table 20: Regression Co-efficients for Annualized CERs in Percent Using CRRA Utility Relative to Equities-

Bonds and Markowitz - Three and Four Year Estimation Periods, and One and Two Regimes - Credit Crisis Excluded

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The dependent variables are the CERs. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns. The excluded months are August 2007 to March 2009.

We repeat the analysis in Table 20, where the credit crisis period is excluded, but using Sharpe ratios as the dependent variable, and these results appear in Table 21. Diversification into commodities remains harmful, and there is some weak evidence that diversification into hedge funds and all seven assets also remains harmful. For both private equities and emerging markets the evidence is mixed - sometimes diversification is harmful, and sometimes it is beneficial. Finally diversification into real estate is generally beneficial when the credit crisis is dropped. Therefore the Sharpe ratio results are broadly similar to those for CERs, but more supportive of harmful diversification.

The two-regime models in Tables 8, 9, 11, 12, 13, 14, 15, 16, 17, and 18 cover the full data period and treat observations in the high and low volatility regimes differently. All the months during the credit crisis, as well as other high volatility and low return periods, are assigned by the regime switching model to the high volatility regime, and this should provide some allowance for unusual periods, such as the credit crisis. In Table 8, with one exception, all the coefficients for two regimes with a three year rolling window are larger than when only one regime is used; suggesting that the use of two regimes has, to some extent, controlled for the credit crisis. However when a four year rolling window is used, although the real estate and private equity coefficients still increase with a two-regime model, all the coefficients for commodities, hedge funds and emerging markets become smaller, i.e. diversification into these three assets becomes more harmful when a two-regime model is used. The Sharpe ratios in Table 9 exhibit the same pattern as in Table 8. When both of the two-regime cases are considered together, the evidence of harmful diversification remains present, indicating that across both three and four year estimation periods the two-regime model has not controlled for the effects of the credit crisis.

	Three Year Estimation Periods - 1997-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	0.835***	0.860***	0.880***	0.614***	0.612***	0.643***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	-0.213***	-0.194***	-0.177***	-0.146***	-0.135***	-0.124***
E&B & Hedge Funds+	-0.035	-0.027	-0.026	0.021	0.020	0.002
E&B & Real Estate	0.023	0.001	-0.008	0.144***	0.127***	0.109***
E&B & Private Equity	-0.075***	-0.070**	-0.059**	0.089***	0.121***	0.141***
E&B & Emerging Markets	0.003	0.028	0.034	0.107***	0.088***	0.102***
All Seven Assets	-0.099***	-0.121***	-0.118***	-0.024	-0.042	-0.047*
18 model dummies, excluding Markowitz which is the comparator						
Adjusted R squared	0.767	0.717	0.724	0.779	0.787	0.782
	Four Year Estimation Periods - 1998-2015					
Regimes	One Regime			Two Regimes		
Risk Aversion	2	5	10	2	5	10
Constant	0.656***	0.716***	0.747***	0.501***	0.553***	0.576***
Equities and Bonds (E&B)	Comparator			Comparator		
E&B & Commodities	-0.098***	-0.120***	-0.132***	-0.182***	-0.265***	-0.303***
E&B & Hedge Funds+	-0.027	-0.026	-0.019	-0.079***	-0.100***	-0.108***
E&B & Real Estate	0.030	0.007	-0.013	0.121***	0.051*	0.020
E&B & Private Equity	-0.008	-0.015	-0.005	0.134***	0.151***	0.159***
E&B & Emerging Markets	0.105***	0.097***	0.089***	-0.059**	-0.052*	-0.043
All Seven Assets	-0.009	-0.024	-0.034	0.041	0.043	0.025
18 model dummies, excluding Markowitz which is the comparator						
Adjusted R squared	0.816	0.812	0.799	0.855	0.851	0.840

Table 21: Regression Co-efficients for Annualized Sharpe Ratios Relative to Equities-Bonds and Markowitz - Three and Four Year Estimation Periods, and One and Two Regimes - Credit Crisis Excluded

+ Hedge fund returns have been de-smoothed using the Geltner (1991, 1993) method. The dependent variables are the Sharpe ratios. Significance at the 10%, 5% and 1% levels is denoted by *, ** and *** respectively for the monthly returns. The excluded months are August 2007 to March 2009.

8. Conclusions

The main question we address is whether investing in alternative assets is beneficial for US investors.

To this end we analyse the effects of adding five alternative assets (commodities, hedge funds, real estate, private equity and emerging markets) to US equity and bond portfolios over the 1997 to 2015 period. We form portfolios using 19 portfolio models, together with different lengths of rolling window, three levels of risk aversion, and with asset returns modelled assuming one or two regimes. Differences between the 19 portfolio models are controlled for by using dummy variable regression. Out-of-sample portfolio performance is measured by both certainty equivalent returns (CERs) and Sharpe ratios. Transactions costs are included when forming portfolios and when evaluating performance. We find that adding alternative assets reduces portfolio performance, i.e. diversification is harmful. The Sharpe ratio results indicate more strongly than the CER results that diversification is harmful. We then investigate three possible causes for this result - estimation errors, transactions costs and non-normal returns; and find that the prime cause of harmful diversification is estimation errors. Estimation errors were much larger during the credit crisis (2007-09), and when this crisis is dropped the finding of harmful diversification disappears for real estate, private equity and emerging markets, but not for commodities and hedge funds. These results indicate that, although diversification into some alternative assets is usually beneficial, it is harmful during periods of extreme market stress, which is probably when investors most need the benefits of diversification. For other alternative assets diversification is generally harmful.

The conclusion of harmful diversification depends on the way investors form portfolios, and how they estimate the inputs to their portfolio model. For example Gao and Nardari (forthcoming) use the forecast combination method of Bates and Granger (1969) to estimate out-of-sample returns, and the dynamic conditional correlation model of Engle (2002) to estimate the out-of sample covariance matrix. If investors form portfolios in a way which substantially reduces the effects of estimation errors, our conclusions may not apply. In addition, although our two performance measures (CERs and Sharpe ratios) are widely used, it is possible that a different measure of portfolio performance would produce different conclusions. We have examined the diversification benefits of five types of alternative asset using returns on investable products representing their aggregate performance. We have not examined the diversification benefits of the individual constituents of these five types of alternative asset; nor have we examined the performance of other types of alternative asset.

As well as our main result, we also found that individual asset and portfolio returns are non-normal; two regimes are present in asset returns and, except for bonds, one regime has higher volatility and lower returns than the other; the use of two regimes makes individual asset returns more normal, supporting the view that asset returns are a mixture of two normal distributions; the use of two regimes leads to much higher levels of trading; estimation errors have a positive correlation with asset weights, and are less of a problem for risk averse investors. Apart from hedge funds, alternative assets have higher estimation errors than equities and bonds; and the errors in predicting portfolio returns are larger for portfolios containing alternative assets. Finally, portfolios with more assets have a worse performance.

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APPENDIX – Portfolio Models

Where applicable, all the portfolio models we use are subject to no short selling constraints, normalization of portfolio weights (weights sum to 1), and upper bounds on asset weights.

Mean-variance (Markowitz). In the Markowitz (1952) mean-variance portfolio model, investors trade-off between the mean and variance of portfolio returns. Portfolio weights are computed by maximizing a quadratic utility function using the sample mean $(\boldsymbol{\mu})$ and covariance matrix $(\boldsymbol{\Sigma})$ of asset returns.

Bayes-Stein shrinkage. The Bayes–Stein model (Jorion, 1986) is designed to tackle estimation errors in mean returns and the covariance matrix of asset returns. The Bayes-Stein model computes the column vector of mean returns $(\boldsymbol{\mu}_{\mathrm{BS}})$ as follows:-

$$\boldsymbol{\mu}_{\mathrm{BS}} = (1-\mathrm{g})\boldsymbol{\mu} + \mathrm{g}\mu_{GMV}\mathbf{1},$$

where $\mathrm{g} = \dfrac{\mathrm{N}+2}{(\mathrm{N}+2) + T(\boldsymbol{\mu}-\mu_{GMV}\mathbf{1})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-\mu_{GMV}\mathbf{1})}$ is the shrinkage factor $(0 \le g \le 1)$, μ_{GMV} denotes the expected return of the minimum variance portfolio when short selling is permitted, $\mathbf{1}$ is a column vector of ones and T is the sample size (estimation period's length). The Bayes–Stein estimator of the covariance matrix of the asset returns $(\boldsymbol{\Sigma}_{\mathrm{BS}})$ is given by:-

$$\boldsymbol{\Sigma}_{\mathrm{BS}} = \left(\frac{T+\varphi+1}{T+\varphi}\right)\boldsymbol{\Sigma} + \frac{\varphi}{T(T+\varphi+1)}\frac{\mathbf{1}\mathbf{1}^{\mathrm{T}}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{1}},$$

where the scalar $\varphi = \dfrac{\mathrm{N}+2}{(\boldsymbol{\mu}-\mu_{min}\mathbf{1})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-\mu_{min}\mathbf{1})}$ represents the precision of the prior distribution of returns. We use the Bayes-Stein estimates $(\boldsymbol{\mu}_{\mathrm{BS}}, \boldsymbol{\Sigma}_{\mathrm{BS}})$.

Bayes diffuse prior. With a diffuse prior $\left(p(\boldsymbol{\mu},\boldsymbol{\Sigma}) \propto |\boldsymbol{\Sigma}|^{-\frac{1}{2}(\mathrm{N}+1)}\right)$ and a normal conditional likelihood, Barry (1974) amongst others, has shown that the predictive distribution of asset returns follows a student's *t*-distribution with mean $\boldsymbol{\mu}$ and variance $(1+1/p)\boldsymbol{\Sigma}$. This portfolio model increases the covariance matrix by $(1+1/p)$, while the historical estimate of mean returns remains unchanged. We use the Bayesian diffuse-prior estimates $(\boldsymbol{\mu},\ (1+1/p)\boldsymbol{\Sigma})$.

Black-Litterman. This model combines the investor's subjective views of expected returns and risks with a benchmark (reference) portfolio. Black and Litterman (1992) compute the posterior column vector of mean returns $(\boldsymbol{\mu}_{\text{BL}})$ as follows:-

$$\boldsymbol{\mu}_{\text{BL}} = \left[(c\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}^{\text{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{P}\right]^{-1}\left[(c\boldsymbol{\Sigma})^{-1}\mathbf{H} + \boldsymbol{P}^{\text{T}}\boldsymbol{\Omega}^{-1}\mathbf{Q}\right],$$

where $\boldsymbol{P}$ is the identity matrix defining the assets involved in each view, $\mathbf{Q}$ is a column vector of the investor's views on asset returns, and c denotes the overall level of confidence in the implied asset returns. The column vector of implied returns for the reference portfolio is computed as $\mathbf{H} = \lambda\boldsymbol{\Sigma}\mathbf{x}^{\text{Reference}}$ where λ is the risk aversion parameter and ($\mathbf{x}^{\text{Reference}}$) denotes the weights of the benchmark (reference) portfolio. For the benchmark portfolio we use the *1/N* portfolio in one model, and the minimum variance portfolio in another model. Following Meucci (2010), the diagonal matrix $\boldsymbol{\Omega}$ that contains the reliability of each view is defined as $\boldsymbol{\Omega} = \frac{1}{\delta}\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^{\mathbf{T}}$, where δ represents the overall level of confidence in the investor's views. We follow Platanakis and Sutcliffe (2017) and set $c = 0.1625$ and $\delta = 1.$ For each estimation period we use the sample mean return of each asset as the investor's view, as in Bessler, Opfer and Wolff (forthcoming) and Platanakis and Sutcliffe (2017). Finally, following Satchell and Scowcroft (2000), we estimate the posterior covariance matrix $(\boldsymbol{\Sigma}_{\text{BL}})$ as follows:-

$$\boldsymbol{\Sigma}_{\text{BL}} = \boldsymbol{\Sigma} + \left[(c\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}^{\text{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{P}\right]^{-1}$$

We use the Black-Litterman estimates $(\boldsymbol{\mu}_{\text{BL}}, \boldsymbol{\Sigma}_{\text{BL}})$.

Minimum variance. For this model we minimize the portfolio variance, subject to the constraints applied to all the optimization models.

Resampled efficient (Michaud). The "re-sampled efficiency" model (Michaud and Michaud, 2008) is based on resampling asset returns to reflect the estimation errors. In each estimation period we estimate the sample means and covariances of asset returns. Then assuming asset returns are drawn from a multivariate normal distribution with these parameters, we draw 20 samples of asset returns, each of which is used to compute an optimal portfolio by maximizing a quadratic utility function. The average of these optimal portfolios gives the overall optimum solution for that estimation period.

1/N with re-balancing. The naive diversification (*1/N*) model assigns a portfolio weight of *1/N* to each risky asset each month (e.g. *1/N* with re-balancing, rather than *1/N*-buy-and-hold).

Risk-parity. This model requires each asset to contribute the same risk to the portfolio variance, see for instance Maillard et al. (2010). By ignoring correlations between assets the asset weights are anti-proportional to their sample variance, and are estimated as follows:-

$$x_i = \frac{1/\sigma_i^2}{\sum_{i=1}^{N}\left(1/\sigma_i^2\right)}, \quad \forall i=1,...,N$$

Reward-to-risk timing. This model, proposed by Kirby and Ostdiek (2012), is based on the reward-to-risk ratio, which is defined as the mean return divided by the sample variance of each asset. This strategy over-weights assets with a higher reward-to-risk ratio (e.g. assets with higher return and lower variance). Portfolio weights are computed as follows:-

$$x_i = \frac{\mu_i^+/\sigma_i^2}{\sum_{i=1}^{N}\left(\mu_i^+/\sigma_i^2\right)}, \quad \forall i=1,...,N,$$

where $\mu_i^+ = max\left(\mu_i,\ 0\right)$ to rule out short selling. In the very rare cases when all asset returns are negative, an equally-weighted portfolio *(1/N)* is used.

CRRA utility. The CRRA (Constant Relative Risk Aversion) utility function is defined as follows:-

$$U\left(W\right) = \frac{1}{1-\lambda}W^{1-\lambda}, \ \text{if } \lambda>0, \lambda \neq 1,$$

where *λ* denotes the investor's relative risk aversion parameter. Following many previous researchers, we use a Taylor series expansion up to the fourth moment to approximate the expected CRRA utility. The Taylor series expansion for the expected CRRA utility function is given by:-

$$E\left(U\left(W\right)\right) \approx \frac{1}{1-\lambda}\bar{W}^{1-\lambda} - \frac{\lambda}{2}\bar{W}^{-(\lambda+1)}\sigma_p^2 + \frac{\lambda\left(\lambda+1\right)}{3!}\bar{W}^{-(\lambda+2)}s_p^3 - \frac{\lambda\left(\lambda+1\right)\left(\lambda+2\right)}{4!}\bar{W}^{-(\lambda+3)}k_p^4$$

where $\bar{W} = 1+\mu_p$, and $\mu_p,\ \sigma_p^2,\ s_p^3,\ k_p^4$ represent respectively the expected return, variance, skewness and kurtosis of the overall portfolio return using sample-based estimates for a given vector of portfolio weights.

Generalized portfolio constraints. Inspired by DeMiguel et al. (2009), the two different versions (lower and upper) of the generalized constraints are:-

$$\mathbf{x} \geq a_{lower}\mathbf{1}, \text{ with } a_{lower} \in [0,\ 1/N] \text{ and}$$

$$\mathbf{x} \leq a_{upper}\mathbf{1}, \text{ with } a_{upper} \in [1/N,\ 1].$$

As in DeMiguel et al. (2009) for the upper and lower bounds we use the middle of their permissible range, and so $a_{lower} = (0+1/N)/2$ and $a_{upper} = (1/N+1)/2.$

Combinations of models. We consider portfolios that are combinations of the *1/N*, minimum-variance and Markowitz tangency portfolios. We apply shrinkage directly to the portfolio weights, and consider optimal linear combinations of asset weights that maximize the expected value of the utility function, as in Kan and Zhou (2007). We employ two different combination strategies; the first combines the *1/N* and minimum-variance portfolios, and the second combines the *1/N*, minimum-variance and Markowitz tangency portfolios. These two strategies can be represented as follows:-

$$\mathbf{x}^{1/\mathrm{N-MV}} = \alpha_1\mathbf{x}^{1/\mathrm{N}} + \alpha_2\mathbf{x}^{\mathrm{MV}}, \qquad \alpha_1,\ \alpha_2 \geq 0$$

$$\mathbf{x}^{1/\mathrm{N-MV-TP}} = \beta_1\mathbf{x}^{1/\mathrm{N}} + \beta_2\mathbf{x}^{\mathrm{MV}} + \beta_3\mathbf{x}^{\mathrm{TP}}, \qquad \beta_1,\ \beta_2,\ \beta_3 \geq 0.$$

When computing $\mathbf{x}^{1/\mathrm{N}}$, $\mathbf{x}^{\mathrm{MV}}$ and $\mathbf{x}^{\mathrm{TP}}$, the no short selling constraint and the normalization of asset weights, but not the upper bound on the sum of alternative asset weights, are applied.The coefficients $\alpha_1,\ \alpha_2$ for $\mathbf{x}^{1/\mathrm{N-MV}}$, and $\beta_1,\ \beta_2,\ \beta_3$ for $\mathbf{x}^{1/\mathrm{N-MV-TP}}$ are chosen to maximize expected mean-variance (quadratic) utility, subject to the constraints we apply to our optimization problems, for both $\mathbf{x}^{1/\mathrm{N-MV}}$ and $\mathbf{x}^{1/\mathrm{N-MV-TP}}$.

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